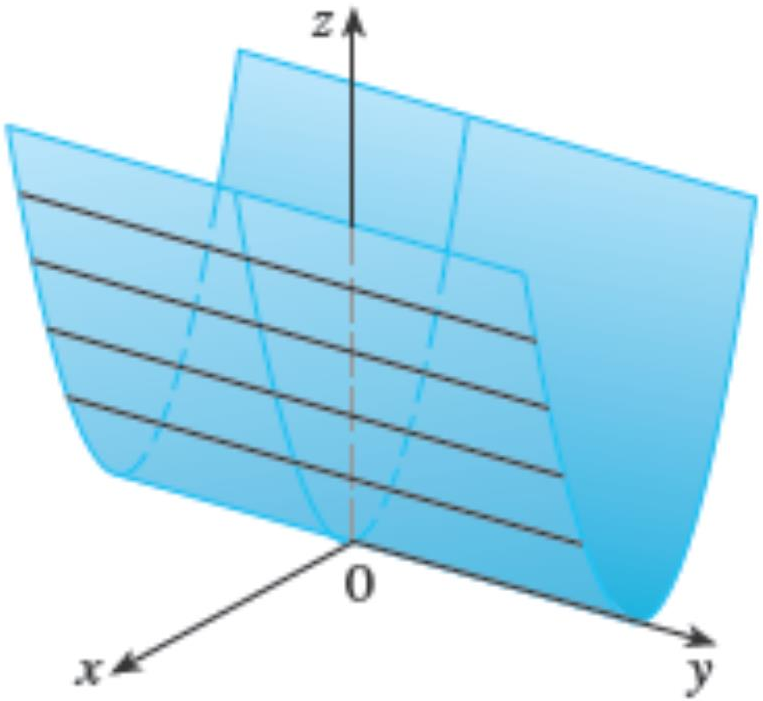
Sec. 12.6:
Cylinders and Quadric Surfaces

What We Will Go Over In Section 12.6

1. Cylinders
2. Traces and quadric surfaces

1. Cylinders

Definition: A cylinder is a surface obtained by starting with a plane curve and extending it in the direction perpendicular to the plane the curve lies in.



Notes:

1) The graph of an equation in 3 variables with one letter missing is a cylinder.
2) Once you figured out the graph of the plane curve, extend it in the direction of the missing letter.

1. Cylinders

Ex 1: Graph the following cylinders…

a) $x^2 + y^2 = 4$

b) $x = z^2$

c) $y = \sin(z)$

d) $yz = 1$

2. Traces and Quadric Surfaces

Definition: A quadric surface is the surface obtained when graphing a 2nd degree equation in x, y, and z. The most general such equation is…

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$$

After a sequence of rotations and translations, all quadratic surface equations can be put into one of the following 2 forms…

$$Ax^2 + By^2 + Cz^2 + J = 0$$

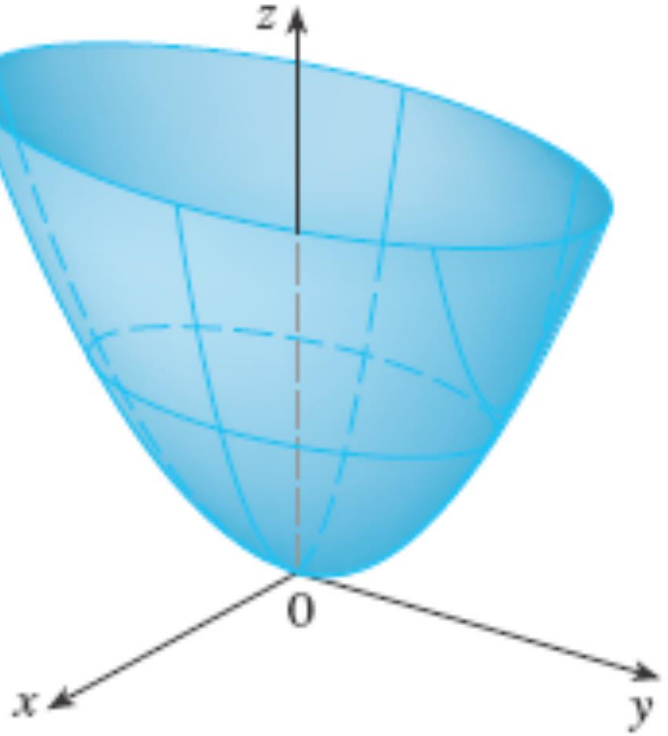
or

$$Ax^2 + By^2 + Iz = 0$$

2. Traces and Quadric Surfaces

Definition: A trace is the intersection of a surface and planes parallel to the *yz*, *xz*, or *xy* planes (or the planes $x=k$, $y=k$, and $z=k$).

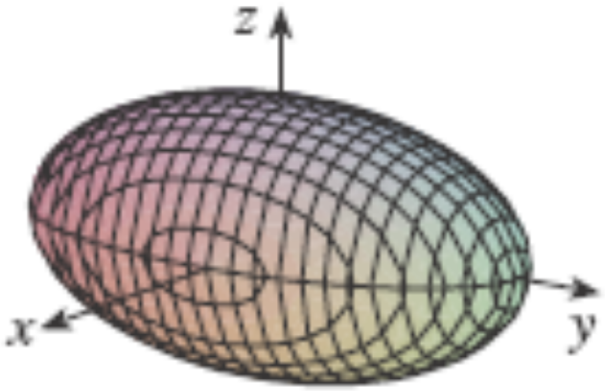
Note: Traces are very useful in visualizing that graph of a surface.



2. Traces and Quadric Surfaces

Ex 2: Find all traces of $\frac{x^2}{2^2}+\frac{y^2}{1^2}+\frac{z^2}{5^2}=1$ and use them to sketch its graph.

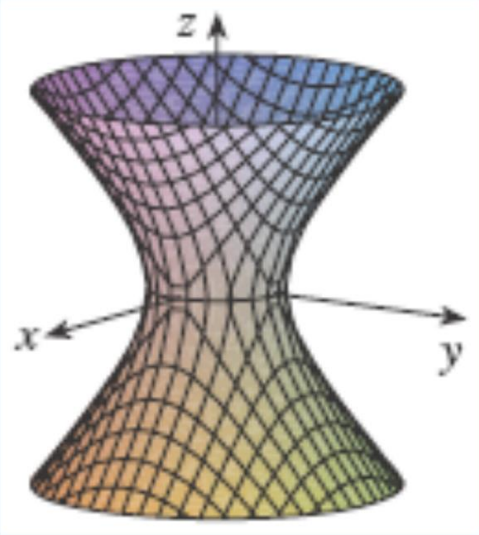
2. Traces and Quadric Surfaces

Surface	Equation
Ellipsoid 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ All traces are ellipses. If $a = b = c$, the ellipsoid is a sphere.

2. Traces and Quadric Surfaces

Ex 3: Find all traces of $\frac{x^2}{2^2} + \frac{y^2}{1^2} - \frac{z^2}{5^2} = 1$ and use them to sketch its graph.

2. Traces and Quadric Surfaces

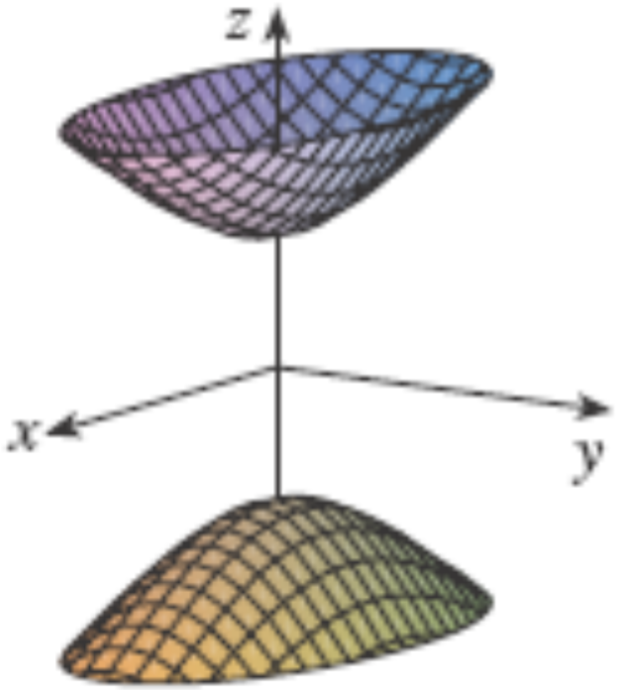
Hyperboloid of One Sheet 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.

2. Traces and Quadric Surfaces

Ex 4: Find all traces of $-\frac{x^2}{2^2}-\frac{y^2}{1^2}+\frac{z^2}{5^2}=1$ and use them to sketch its graph.

2. Traces and Quadric Surfaces

Hyperboloid of Two Sheets



$$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

Horizontal traces in $z=k$ are ellipses if $k>c$ or $k<-c$.

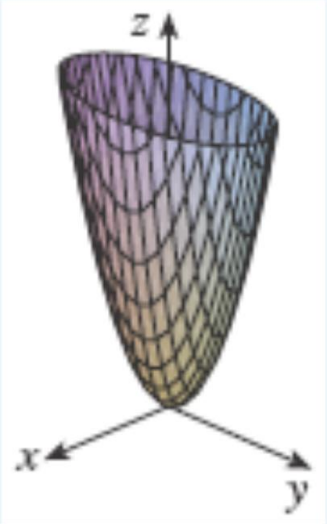
Vertical traces are hyperbolas.

The two minus signs indicate two sheets.

2. Traces and Quadric Surfaces

Ex 5: Find all traces of $z = 2x^2 + y^2$ and use them to sketch its graph.

2. Traces and Quadric Surfaces

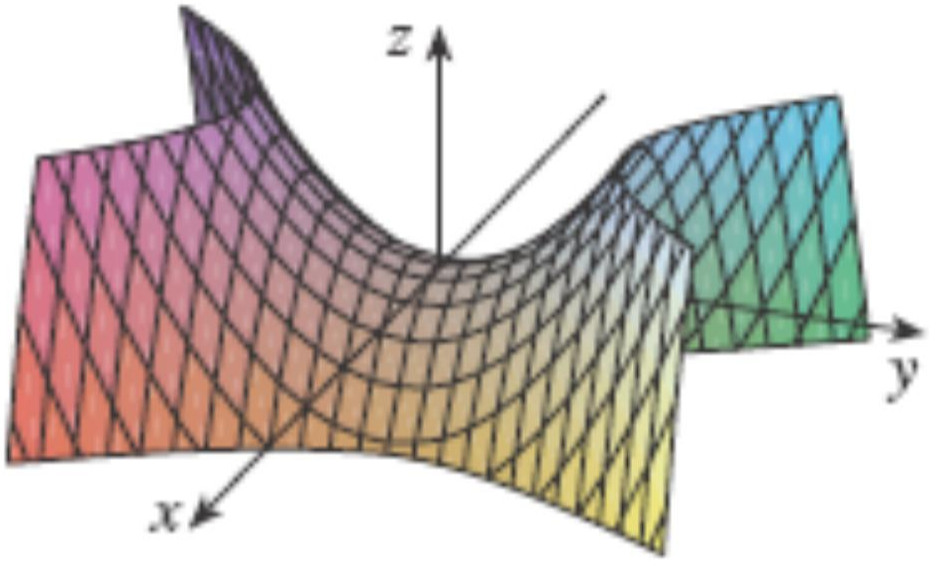
Elliptic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.

2. Traces and Quadric Surfaces

Ex 6: Find all traces of $z = 2x^2 - y^2$ and use them to sketch its graph.

2. Traces and Quadric Surfaces

Hyperbolic Paraboloid



$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

Horizontal traces are hyperbolas.

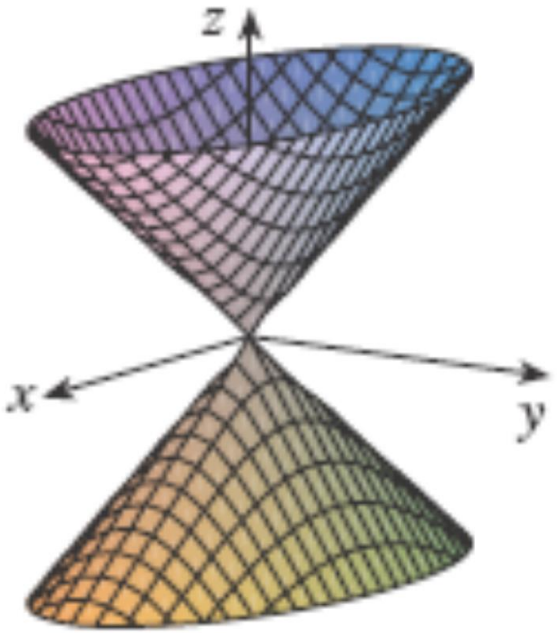
Vertical traces are parabolas.

The case where $c < 0$ is illustrated.

2. Traces and Quadric Surfaces

Ex 7: Find all traces of $z^2 = 2x^2 + y^2$ and use them to sketch its graph.

2. Traces and Quadric Surfaces

Surface	Equation
Cone 	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.

2. Traces and Quadric Surfaces

Ex 8: Find all traces of $-\frac{x^2}{2^2}+\frac{y^2}{1^2}-\frac{z^2}{5^2}=1$ and use them to sketch its graph.

2. Traces and Quadric Surfaces

Ex 9: Write the following equation in standard form, then sketch its graph.

$$2x^2 - 8y^2 + 2z^2 + 12x + 64y - 108 = 0$$